

Applied Statistics Comprehensive Examination**Statistical Theory I & II**

- Calculators are not permitted on this part of the examination.
- Give complete explanations, showing all work and simplifying answers.
- The numbers in parentheses indicate point values for each question.

- (20) 1. A fair die is rolled once producing the number N . Now that die is rolled N times. Find the expected value of the sum of those N rolls.
- (20) 2. Suppose X and Y are independent random variables, with X uniformly distributed on $[0,2]$ and Y uniformly distributed on $[0,3]$. Find $P(X + Y \leq 1)$.
- (60) 3. Let X be a random variable from a distribution with probability density function

$$f_X(x) = \begin{cases} \theta x^{\theta-1} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$.

- (5) a. Consider testing $H_0: \theta = 1$ versus $H_a: \theta = 2$ based on a single observation where H_0 is rejected if $X > .8$. Find the probability of a Type II Error.
- (10) b. Consider testing $H_0: \theta = 1$ versus $H_a: 1 < \theta \leq 3$ based on a single observation where H_0 is rejected if $X > .8$. Find the power function for the test if $1 \leq \theta \leq 3$.
- (20) c. Consider testing $H_0: \theta = 1$ versus $H_a: \theta > 1$ based on the random sample X_1, X_2, X_3 , where H_0 is rejected if at least 2 of the sample observations are larger than .8. Find the level of significance of this test.
- (10) d. Let 0.1 and 0.3 be a random sample from this distribution. Find $\tilde{\theta}$, the method of moments estimate for θ .
- (15) e. Let 0.1 and 0.3 be a random sample from this distribution. Find $\hat{\theta}$, the maximum likelihood estimate of θ . Verify that $\hat{\theta}$ is a maximum.