Name _____ Spring, 2015

Applied Statistics Comprehensive Examination

Statistical Theory I & II

- Calculators are not permitted on this part of the examination.
- The numbers in parentheses indicate point values for each question.
- Give complete explanations for all answers.

(25) 1. Box 1 contains 2 white cards and 3 black cards. Box 2 contains 5 white cards and 2 black cards. Two cards are drawn at random, without replacement, from Box 1 and put into Box 2. Next, two cards are drawn at random, without replacement, from Box 2. Suppose the two cards drawn from Box 1 were both the same color, and the two cards drawn from Box 2 were both black. Find the probability that the two cards drawn from Box 1 were both black.

(40) 2. Let X have the probability density function

$$f_X(x) = \begin{cases} \dfrac{2\theta^2}{x^3} & x \geq \theta \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$.

(10) *a.* If k denotes the median of this distribution, find the value of k.

(15) *b.* It is claimed that the variance of X is $\frac{\theta^2}{4}$. Give a complete discussion of whether this is or is not true.

(15) *c.* Let 1, 10, 4 be a random sample from a population having this distribution. Find $\hat{\theta}$, the maximum likelihood estimate of θ.

(35) 3. Let X have the probability density function

$$f_X(x) = \begin{cases} \theta & 0 < x \leq \dfrac{1}{\theta} \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$.

(15) *a.* Let $\frac{1}{4}$, $\frac{1}{2}$ be a random sample from a population having this distribution. Find $\tilde{\theta}$, the method of moments estimate of θ.

(20) *b.* Suppose we are testing $H_0\colon \theta = 2$ versus $H_a\colon \theta = 3$ based on a single observation X and critical region $X < \frac{1}{10}$.

(5) *i.* Find the probability of a Type I Error.

(5) *ii.* Find the probability of a Type II Error.

(10) *iii.* Find the power function for this test.