Spring, 2006

Applied Statistics Comprehensive Examination

Statistical Theory I & II

(25) 1. A fair coin is tossed until the same result occurs twice in a row. Find the probability that this occurs on the k^{th} toss. Verify that your answer is a probability mass function.

(25) 2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with mean μ and variance $\sigma^2 > 0$. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Find the bias of $(\bar{X})^2$ as an estimator of μ^2.

(25) 3. Let $X_1 = \frac{1}{4}$ and $X_2 = \frac{9}{16}$ be a random sample from a population with probability density function

$$f_X(x) = \begin{cases} 1 - 8\lambda + 12\lambda\sqrt{x} & \text{if } 0 \le x \le 1, \; -\frac{1}{4} \le \lambda \le \frac{1}{8} \\ 0 & \text{otherwise} \end{cases}$$

with mean $\mu = \frac{1}{2} + \frac{4}{5}\lambda$.

a. Find $\tilde{\lambda}$, the method of moments estimate of λ.
b. Find $\hat{\lambda}$, the maximum likelihood estimate of λ.

(25) 4. Consider a population with probability density function

$$f_X(x) = \begin{cases} 1 - \theta^2 & \text{if } 0 < x < \frac{1}{1-\theta^2}, \; 0 \le \theta < 1 \\ 0 & \text{otherwise} \end{cases}$$

We want to test $H_0 : \theta = \frac{1}{2}$ versus $H_1 : \theta = \frac{3}{4}$ using a single observation X and critical region $X > 1$.

a. Find the level of significance of this test.
b. Find the probability of making a Type II Error.
c. Find the power function of this test.
d. Using the answer to part *c*, find α and β.