Name ________________________ Fall, 2007

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.
Answers to all questions require complete explanations to receive full credit.

(20) 1. A factory has three different assembly lines that manufacture the same product. Line A accounts for 50%, Line B for 30% and Line C for 20%. The rates of defective products are 10% for Line A, 20% for Line B, and 40% for Line C. Suppose that one product is selected at random from the factory's output.

a. What is the probability that the product is defective?
b. If the product is defective, what is the probability that it came from Line C?

(20) 2. Suppose X_1 and X_2 are independent random variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 respectively. If $Y = X_1X_2$, find the mean and variance of Y.

(30) 3. Let X_1, X_2 and X_3 be a random sample from a population with probability density function

$$f_X(x) = \begin{cases} (\theta+1)x^\theta & \text{if } 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$.

a. Find the method of moments estimate of θ.
b. Find the maximum likelihood estimate of θ.

(30) 4. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with probability mass function

$$p_X(x) = \begin{cases} p(1-p)^x & \text{if } x = 0,\ 1,\ \ldots \\ 0 & \text{elsewhere} \end{cases}$$

where $0 < p < 1$.

a. For $n = 1$, find the power function of the test with critical region $x \le 1$.

b. For any positive integer n, using the Neyman-Pearson Lemma, find the best critical region when testing $H_0\colon p = \frac{1}{2}$ versus $H_a\colon p = \frac{3}{4}$. Simplify your answer.

c. Determine if the critical region found in part *b* is best when testing $H_0\colon p = \frac{1}{2}$ versus $H_a\colon p > \frac{1}{2}$, and give reasons for your answer.