Name ______________________________ Spring, 2007

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.

(20) 1. Suppose X_1, X_2, and X_3 are independent random variables, where X_1 has a normal distribution with mean 2 and variance σ^2, X_2 has a normal distribution with mean 1 and variance σ^2 and X_3 has a normal distribution with mean 2 and variance σ^2. Let

$$W = c\left[\frac{4(X_1 - 2)^2}{(X_2 - 1)^2 + (X_3 - 2)^2}\right]$$

Find c such that W will have an $F(1, 2)$ distribution. Simplify your answer.

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Name ______________________________

(25) 2. Let X_1, X_2, and X_3 be independent Poisson random variables with means θ, 2θ and 3θ, respectively. Derive the maximum likelihood estimator of θ. Simplify your answer.

Name ______________________________

(25) 3. Let $X_1,\ X_2,\ \ldots,\ X_n$ be a random sample from a population with probability density function

$$f_X(x) = \begin{cases} \theta(4-x)^{\theta-1} & \text{if } 3 \le x \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

where $\theta > 0$.

a. Consider $H_0: \theta = \theta_0$ versus $H_a: \theta = \theta_a$ where $\theta_a > \theta_0$. Find the best critical region (Neyman-Pearson Lemma) for this test. Simplify your answer.

b. If there is only one observation X from this population, find the best critical region with $\alpha = 0.05$ when testing $H_0: \theta = 1$ versus $H_a: \theta = 2$.

Name ____________________

(30) 4. Let T_1 and T_2 be unbiased estimators of a population parameter θ, each estimator based upon a different independent random sample. Let T_1 and T_2 have variances σ_1^2 and σ_2^2, respectively.

a. Show that $T = \lambda T_1 + (1 - \lambda)T_2$ is also an unbiased estimator of θ.

b. Find the value of λ that minimizes the variance of T.

c. Discuss how the solution to the problem in part *b* would change if T_1 and T_2 were based on the *same* random sample. (It is not necessary to find this new solution. Just indicate what would have to be considered.)