

**Applied Statistics Comprehensive Examination****Statistical Theory I & II**

Calculators are not permitted on this part of the examination.

**Give complete explanations for all answers.**

- (25) 1. In a college town of 500 students, it is known that 60% of the students are women. In addition, it is known that 1% of the women and 10% of the men are taller than 6 feet.
- (10) *a.* If a randomly selected student is taller than 6 feet, find the probability that the student is a woman. It is not necessary to simplify your answer.
- (15) *b.* If a team of three men is selected at random without replacement, find the probability that the team has at most one man who is taller than 6 feet. It is not necessary to simplify your answer.

- (25) 2. Let  $X$  be a random variable with probability mass function

$$p_X(x) = \begin{cases} p(1-p)^{(x-1)} & \text{if } x = 1, 2, 3, \dots \\ 0 & \text{elsewhere} \end{cases}$$

with  $0 < p < 1$ .

- (20) *a.* Derive the maximum likelihood estimate of  $p$  based on three independent observations: 1, 1, 3.
- (5) *b.* Find the maximum likelihood estimate of  $p^2$  based on the same three independent observations 1, 1, 3.
- (25) 3. Let  $X$  and  $Y$  be independent random variables where:  
 $E(X) = 1$ ,  $E(X^2) = 2$ ,  $E(X^3) = 4$ ,  $E(X^4) = 8$ ,  $E(Y) = 3$  and  $E(Y^2) = 10$ .
- (10) *a.* Find  $Var(X - Y)$ .
- (15) *b.* Find  $Var(X^2Y)$ .

(25) 4. Let random variable  $X$  have probability density function

$$f_X(x) = \begin{cases} \frac{1}{2\theta x} & \text{if } e^{-\theta} < x < e^{\theta} \\ 0 & \text{elsewhere} \end{cases}$$

with  $\theta > 0$ . We wish to test  $H_0: \theta = 5$  versus  $H_a: \theta = 6$  using a single observation  $X$  and the critical region  $x \geq e^4$ .

(7) *a.* Find  $\alpha$  for this test.

(7) *b.* Find the probability of the Type II Error.

(11) *c.* Find the power function for this test.