Spring, 2008

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.
Answers to all questions require complete explanations to receive full credit.

(25) 1. Let X and Y be random variables with joint probability density function $f_{X,Y}(x,y)$. Prove that if X and Y are independent then $E(XY) = E(X)E(Y)$.

(25) 2. Suppose that the random variables U, V and W have mean vector (1, 2, 3) and variance-covariance matrix

$$\Sigma = \begin{pmatrix} 4 & 1 & 1 \\ 1 & 5 & 2 \\ 1 & 2 & 6 \end{pmatrix}$$

Find the correlation between $X = U + 2V + 1$ and $Y = V - W$.

(25) 3. Let X_1, X_2 be a random sample from a normal distribution with mean 3 and variance θ. If the sample values are 4 and 0, find $\hat{\theta}$, the maximum likelihood estimator of θ.

(25) 4. Consider a random sample $X_1, X_2, \ldots, X_{25}$ from a normal distribution with mean μ and variance 100. We want to test H_0: $\mu = 80$ versus H_a: $\mu = 83$. Consider the test with critical region $\bar{X} \geq 83$ where

$$\bar{X} = \frac{1}{25}\sum_{i=1}^{25} X_i$$

Let $\Phi(z)$ denote the distribution function of a standard normal random variable. Use $\Phi(z)$ in any of the following answers whenever it is necessary.

(9) *a.* Find the power function of the test. If it is not possible to find the power function, state why.

(6) *b.* Find α for this test. If it is not possible to find α, state why.

(5) *c.* Find the probability of the Type II Error. If it is not possible to find the probability of the Type II Error, state why.

(5) *d.* Consider the critical region $\bar{X} \geq k$ for some constant k. Discuss if there is any impact on β if you attempt to make α smaller. Discuss if there is a strategy for making both α and β smaller simultaneously. Reasons must be given to support your answers.