

Applied Statistics Comprehensive Examination**Statistical Theory I & II**

Calculators are not permitted on this part of the examination.

Give complete explanations for all answers.

- (25) 1. Consider a fair die whose sides are 1, 2, 2, 3, 3, 3. After rolling this die, a fair coin is tossed the number of times indicated on the die. Find the probability that the die showed a 3 given that the results of the tosses were all heads.

- (15) 2. Let X and Y have the joint probability mass function

$$p_{X,Y}(x,y) = \begin{cases} \frac{1}{6} & \text{if } x = 1, 2, 3; y = 2, 3 \\ 0 & \text{otherwise} \end{cases}$$

Let $U = X + Y$.

- (10) *a.* Find the probability mass function of U .

- (5) *b.* Find the moment generating function of U .

- (25) 3. A box contains 100 chips, r red and $100 - r$ white. Suppose 20 chips are chosen at random without replacement. We want to test $H_0: r = 30$ versus $H_a: r > 30$ using the critical region: at least 10 red chips are obtained. Do not simplify your answers.

- (15) *a.* Find the power function of this test.

- (10) *b.* If $r = 40$, find an expression for $P(\text{Type II Error})$.

- (35) 4. Consider a random sample of size 5 from a population with probability density function

$$f_X(x) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

with $\theta > 0$.

- (15) *a.* Find the method of moments estimator of θ .

- (20) *b.* Find the maximum likelihood estimator of θ , and verify it is a relative maximum.