

Applied Statistics Comprehensive Examination**Statistical Theory I & II**

Calculators are not permitted on this part of the examination.

Give complete explanations for all answers.

- (20) 1. Let X have the probability density function

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$. If the median of X is $\frac{1}{3}$, find λ .

- (20) 2. Consider the random sample X_1, \dots, X_5 from a population with mean 0 and variance σ^2 . Suppose $S = X_1 + X_2 + X_3$ and $T = X_2 + X_3 + X_4 + X_5$. Find the correlation coefficient ρ_{ST} .

- (30) 3. Suppose 4, 8 is a random sample from a normal population with mean 3 and variance σ^2 . Derive the maximum likelihood estimate of σ^2 .

- (30) 4. Let X_1, \dots, X_n be a random sample from

$$f_X(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$.

- (15) a. Consider testing $H_0: \theta = \frac{1}{2}$ vs. $H_a: \theta = \frac{1}{3}$. Derive the best test based on this random sample.
- (15) b. Consider testing $H_0: \theta = \frac{1}{2}$ vs. $H_a: \theta < \frac{1}{2}$. Give a complete explanation as to why there is or why there is not a uniformly most powerful test. If there is such a test, describe it completely.