

**Applied Statistics Comprehensive Examination****Statistical Theory I & II**

Calculators are not permitted on this part of the examination.

**Give complete explanations for all answers.**

- (25) 1. A box contains 1 red chip, 2 white chips and 3 blue chips. Three chips are chosen at random without replacement. Let  $X$  denote the number of white chips and  $Y$ , the number of blue chips. Give the definition of independence of  $X$  and  $Y$  in this context. Prove that  $X$  and  $Y$  are not independent.

- (25) 2. A shipping company charges customers \$1 plus an additional  $D$  dollars to ship a package weighing  $W$  pounds, where  $W$  is a random variable with probability density function

$$f_W(w) = \begin{cases} 2w & \text{if } 0 < w < 1 \\ 0 & \text{otherwise} \end{cases}$$

Assume that  $D = 3W^2$  and that  $T$  is the total cost to the customer.

- (10) a. Find  $E(T)$

- (15) b. Find  $Var(T)$ .

- (25) 3. Consider a population with probability mass function

$$p_X(x) = \begin{cases} \frac{1}{2} + \theta & \text{if } x = 0 \\ \frac{1}{2} - \theta & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

where  $\theta$  is an unknown parameter with  $-\frac{1}{2} < \theta < \frac{1}{2}$ . The random sample 1, 0 is drawn from this population.

- (10) a. Find the method of moments estimate of  $\theta$ .

- (15) b. Find the maximum likelihood estimate for  $\theta$ . Verify that it is a maximum.

- (25) 4. Let  $X_1, X_2$  be a random sample from a Poisson distribution with mean  $\theta$ . Consider testing  $H_0: \theta = 5$  versus  $H_a: \theta = 3$  with test statistic  $Y = X_1 + X_2$ .

- (10) a. Find the probability of a Type I error if the critical region is  $Y < 2$ .

- (15) b. Find the form of the best critical region when testing  $H_0$  versus  $H_a$ , and simplify your answer.