Applied Statistics Comprehensive Examination
Regression & Linear Models

1. (30 Points) A multiple regression model contains three continuous explanatory variables: X_1, X_2, and X_3. The variance inflation factors (VIFs) for each of the three variables are:

Variable	VIF
X_1	11.7
X_2	1.2
X_3	11.1

   (a) Discuss the presence of multicollinearity in the model.

   (b) Describe how the VIF for X_1 was calculated.

   (c) Describe the potential problems if both X_1 and X_3 are present in the final model.

2. (20 Points) Discuss how each of the following model selection techniques is used to select the best subset model from a larger set of possible explanatory variables:

   (a) Mallow's C_p Statistic

   (b) Stepwise Regression

3. (30 points) An experiment to compare three treatments was run in a completely randomized manner. The following data were obtained.

Treatment	Values				Sum
A	19	20	15		54
B	12	18	15	11	56
C	10	13	7		30

   (a) Assuming the usual fixed effects model, find the normal equations.

   (b) Find a solution to the normal equations.

   (c) Using $\alpha = 0.05$, test for a difference between the average of the means for treatments A and B and the mean for treatment C. You may use the fact that $\hat{\sigma}^2 \approx 8.857$.

4. (20 points) An experiment was run to study the effect of two factors A and B on a response variable. The data are tabled below. Assume that the data will be analyzed using the model $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \epsilon_{ijk}$, $i = 1, 2, 3$, $j = 1, 2$, where α_i and β_j are fixed main effects, γ_{ij} is an interaction effect, and the values $\{\epsilon_{ijk}\}$ are i. i. d. $N(0, \sigma^2)$ random variables.

	B	
A	1	2
1	0,4	2,6
2	4,8	7,7
3	8,10	9

(a) Find the population marginal mean (in terms of the parameters) for each level of B.

(b) Find the least squares mean for each level of A.

(c) Classify each expression as estimable or not estimable, and justify your answer. (i) $\beta_1 - \beta_2$ (ii) $\gamma_{11} - \gamma_{12} - \gamma_{21} + \gamma_{22}$