Name ______________________________ Spring, 2010

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.
Give complete explanations for all answers.

(25) 1. Five cards are chosen at random without replacement from a pack of 7 cards numbered 1 through 7. Let X denote the minimum number from those 5 cards. Find the probability mass function of X.

(25) 2. Let $X_1,\ X_2$ be a random sample from a distribution with mean μ and variance σ^2. Find k so that

$$k\left[(X_1 - X_2)^2\right]$$

is an unbiased estimator of σ^2.

(25) 3. Suppose that 4, 6 is a random sample from the uniform distribution on the interval $(\theta, 2\theta)$, where $\theta > 0$.

(15) *a.* Find the method of moments estimator of θ.

(10) *b.* Write down the likelihood function in simplest form.

(25) 4. Consider a population with probability density function

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose we consider testing H_0: $\lambda = 2$ vs. H_a: $\lambda = 1$.

(5) *a.* Let X_1 denote a sample of size one. Find β if the critical region is $X_1 < 1$.

(10) *b.* Let X_1 denote a sample of size one. Find the power function of the test with critical region: $X_1 < 1$.

(10) *c.* Let X_1, X_2 denote a random sample of size two. Find the form of the best critical region for testing H_0 versus H_a, and simplify your answer.