Name ______________________________ Fall, 2011

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.

Give complete explanations for all answers.

(20) 1. Let X and Y be random variables with probability density function

$$f_{X,Y}(x,y) = \begin{cases} cx & \text{if } 0 < x < 2, \quad 0 < y < 2-x \\ 0 & \text{otherwise} \end{cases}$$

(10) *a.* Find c.

(10) *b.* Using the definition of independence, determine whether X and Y are independent.

(30) 2. Let X_1, X_2, X_3 be independent Bernoulli random variables with mean p and variance $p(1-p)$. Suppose we estimate p with $\hat{p} = \frac{1}{3}\sum_{i=1}^{3} X_i$.

(5) *a.* Find the bias of $\hat{p}$ as an estimate of p.

(10) *b.* Find the variance of $\hat{p}$.

(15) *c.* Find the bias of $\hat{p}(1-\hat{p})$ as an estimate of $p(1-p)$.

(20) 3. Let 2, 3, 7 be a random sample from a population with probability density function

$$f_X(x) = \begin{cases} \dfrac{2(\theta - x)}{\theta^2} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$.

(15) *a.* Find the method of moments estimate of θ.

(5) *b.* Find the likelihood function.

(30) 4. Consider testing H_0: $f_X(x) = f_0(x)$ versus H_a: $f_X(x) = f_a(x)$ based on a sample of size 1 where

$$f_0(x) = \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad f_a(x) = \begin{cases} 2-2x & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Consider the critical region $X > \frac{1}{2}$.

(7) *a.* Find α.

(8) *b.* Find the power if H_a is true.

(15) *c.* Find the form of the best critical region when testing H_0 versus H_a. Simplify your answer.