Name ______________________________ Spring, 2011

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.
Give complete explanations for all answers.

(25) 1. Let X be a random variable with probability mass function

$$p_X(x) = \begin{cases} \dfrac{x}{10} & \text{if } x = 1, 2, 3, 4 \\ 0 & \text{otherwise} \end{cases}$$

Let X_1, X_2, X_3 be a random sample from this distribution. Find $P(X_1 < X_2 < X_3)$.

(25) 2. Suppose U_1, U_2 and U_3 are independent random variables, distributed uniformly on $(0, \theta)$, where $\theta > 0$ is an unknown parameter. Let $Y_{(3)} = \max\{U_1, U_2, U_3\}$. Find the probability that $Y_{(3)} > \theta/2$.

(25) 3. Let 1, 7, 4 be a random sample from a population with probability density function

$$f_X(x) = \begin{cases} \dfrac{1}{2\theta} e^{-\frac{x}{2\theta}} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

with $\theta > 0$, having mean 2θ and variance $4\theta^2$.

(10) *a.* Find a method of moments estimate of θ^2.

(15) *b.* Find the maximum likelihood estimate of θ^2. Verify that it is a maximum.

(25) 4. Consider a population with probability density function

$$f_X(x) = \begin{cases} (\theta + 1)x^{\theta} & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta \geq 0$. Suppose we want to test the hypothesis H_0: $\theta = 2$ vs. H_a: $\theta = 3$ using a single observation X and critical region $X < \frac{1}{2}$.

(5) *a.* Find the probability of making a Type II Error.

(10) *b.* Find the power function of the test.

(10) *c.* Derive the form of the best critical region using the Neyman-Pearson Lemma for the random sample X_1, X_2, and simplify your answer.