

Applied Statistics Comprehensive Examination
Statistical Theory I & II

Calculators are not permitted on this part of the examination.
Give complete explanations for all answers.

- (25) 1. In a certain experiment, the probability of obtaining a score less than x is

$$G(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^3 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

Find the probability of obtaining a score greater than the mean score.

- (20) 2. If the random variable X has the probability density function

$$f_X(x) = \begin{cases} \frac{3x^2}{2} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and $Y = e^X$, find $f_Y(y)$ the probability density function of Y .

- (15) 3. A random sample of size 7 is drawn from an infinite population with mean 0 and variance 1. Denote the mean of the first five values by \bar{x}_5 and the mean of the last five values by \bar{y}_5 . Find the variance of $\bar{x}_5 - \bar{y}_5$.
- (40) 4. For random variable X and any value of θ , consider a population with probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\theta)^2} \quad \text{with } -\infty < x < \infty$$

- (20) a. Find the best critical region using the Neyman-Pearson Lemma for a random sample of size n when testing $H_0: \theta = 2$ versus $H_a: \theta = 1$. Simplify your work.
- (20) b. If the observations of a random sample are -2 , -5 and 8 , find the maximum likelihood estimate of θ . Verify that your answer is a maximum.