Name ______________________________ Spring, 2012

Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination.
Give complete explanations for all answers.

(25) 1. Assume that X has a uniform distribution on $(0, 1)$.

(10) *a.* Find the probability that one of two independent observations of X falls in the interval $(\frac{1}{4}, \frac{1}{2})$.

(15) *b.* Let $Y = \frac{1}{2}\ln(X)$. Find $f_Y(y)$, the probability density function of Y, and verify that it is a probability density function.

(20) 2. Let X have a Poisson distribution with parameter λ. If $W = \dfrac{2X + 3X^2}{5}$, determine the bias of W as an estimate of λ.

(55) 3. Consider random samples from the population with probability density function

$$f_X(x) = \begin{cases} \theta^4 x e^{-\theta^2 x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$, $E(X) = \dfrac{2}{\theta^2}$ and $Var(X) = \dfrac{2}{\theta^4}$.

(10) *a.* If 16, 20 is a random sample, find $\widetilde{\theta^2}$, the method of moments estimate of θ^2.

(15) *b.* If 6, 10 is a random sample, find $\hat{\theta}$, the maximum likelihood estimate of θ.

(7) *c.* Using the results of part 3*b*, prove that $\hat{\theta}$ is a maximum.

(15) *d.* Consider testing H_0: $\theta = 1$ versus H_a: $\theta = \frac{1}{2}$. Derive the form of the best critical region when testing H_0 versus H_a for the random sample X_1, X_2, X_3, giving your answer in simplified form.

(8) *e.* Consider testing H_0: $\theta = 1$ versus H_a: $\theta = \frac{1}{2}$. Using a single observation X and critical region $X < 1$, find α.