(25) 1. Suppose we have three fair dice, A, B and C. A has 4 red sides and 2 white sides, B has has 3 red sides and 3 white sides, and C has two red sides and 4 white sides.

(10) *a.* Suppose all three dice are tossed at the same time. Let X equal the number of red sides. Find $P(X \leq 1)$.

(15) *b.* Suppose a die is chosen at random and rolled twice. If the two rolls produced one red side and one white side, find the probability that die B was chosen.

(25) 2. Let X and Y be independent random variables, each with probability density function

$$f_X(w) = f_Y(w) = \begin{cases} \frac{1}{4\theta} & \text{for } -2\theta < w < 2\theta \\ 0 & \text{otherwise} \end{cases}$$

with $\theta > 0$. If $\text{Var}(XY) = \frac{64}{9}$, find the value for θ.

(30) 3. Let .5 and .7 be a random sample from a population with probability density function

$$f_X(x) = \begin{cases} (\theta+1)x^\theta & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$.

(10) *a.* Find $\tilde{\theta}$, the method of moments estimate of θ.

(20) *b.* Find $\hat{\theta}$, the maximum likelihood estimate of θ. Verify that it is a maximum.

(20) 4. For random variable X and unknown parameter θ, with $\theta > 0$, consider a population with probability mass function

$$p_X(x) = \frac{e^{-\theta}\theta^x}{x!}, \text{ if } x = 0,\ 1,\ \ldots$$

Find the best critical region using the Neyman-Pearson Lemma for random sample $X_1,\ X_2,\ \ldots, X_{10}$ when testing H_0: $\theta = .7$ versus H_a: $\theta = .5$.