

theory

- (30) 1. A fair die has 1 side that is red, 2 sides that are white and 3 sides that are blue. It is tossed three times.
- (10) a. Find the probability that a color occurs at least twice.
 - (10) b. Find the probability that white occurs at least twice.
 - (10) c. Find the probability that white occurs at least twice if a color occurs at least twice.

- (20) 2. Let X and Y be random variables with probability density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{4} & \text{if } 0 < x < 2, \quad 0 \leq y \leq 2x \\ 0 & \text{otherwise} \end{cases}$$

Find $E(XY)$.

- (50) 3. The probability density function of random variable X is

$$f_X(x) = \begin{cases} 2(1 - \theta)x + \theta & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

with $0 \leq \theta \leq 2$, where $E(X) = \frac{4 - \theta}{6}$ and $Var(X) = \frac{1}{36}(2 + 2\theta - \theta^2)$.

- (10) a. Based on the single observation, $X = \frac{1}{2}$, find the method of moments estimate of θ .
- (10) b. Based on the single observation, $X = \frac{1}{4}$, find the maximum likelihood estimate of θ .
- (30) c. Consider testing $H_0: \theta = \frac{3}{2}$ versus $H_a: \theta = \frac{1}{2}$.
 - (10) i. Based on a sample of size 1, find the form of the best critical region.
 - (10) ii. For critical region: $x < c$, find an expression for α in terms of c .
 - (10) iii. For critical region: $x > c$, find an expression for β in terms of c .