

Applied Statistics Comprehensive Examination**Statistical Theory I & II**

Calculators are not permitted on this part of the examination.

There must be complete explanations for all answers.

- (25) 1. A fair roulette wheel has 4 slots numbered 1, 2, 3 and 4. Let X denote the outcome of the first spin and Y the outcome of the second spin.

(5) *a.* Find $P(X = Y)$.

(10) *b.* Find $P(X > Y)$.

(10) *c.* Find $E(X - Y|X > Y)$.

- (15) 2. The joint probability density function of random variables X and Y is

$$f_{X,Y}(x,y) = \begin{cases} x^2 + xy & 0 < y < 2x, 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the mean of X .

- (60) 3. Let X be a random variable with probability mass function

$$p_X(x) = p(1-p)^{x-1} \text{ where } x = 1, 2, \dots \text{ and } 0 < p < 1.$$

The mean and variance of X are $\frac{1}{p}$ and $\frac{1-p}{p^2}$, respectively.

- (30) *a.* Consider a random sample with observations 3, 8, 1.

(10) *i.* Find \tilde{p} , the method of moments estimate of p .

(20) *ii.* Find \hat{p} , the maximum likelihood estimate of p . Verify that \hat{p} is a maximum.

- (20) *b.* Consider a random sample with observations X_1, X_2, \dots, X_{100} . Find the best test using the Neyman-Pearson Lemma if you are testing $H_0: p = 0.2$ versus $H_a: p = 0.6$.

- (10) *c.* For a sample of size 1, suppose you are testing $H_0: p = 0.2$ versus $H_a: p = 0.6$ with critical region $X < k$. Find α if $k = 3$.