Applied Statistics Comprehensive Examination

Statistical Theory I & II

Calculators are not permitted on this part of the examination. Give complete explanations for all answers.

(25) 1. A box contains 1 red chip, 2 white chips and 3 blue chips. Three chips are chosen at random without replacement. Let $X$ denote the number of white chips and $Y$, the number of blue chips. Give the definition of independence of $X$ and $Y$ in this context. Prove that $X$ and $Y$ are not independent.

(25) 2. A shipping company charges customers $1 plus an additional $D$ dollars to ship a package weighing $W$ pounds, where $W$ is a random variable with probability density function

$$f_W(w) = \begin{cases} 2w & \text{if } 0 < w < 1 \\ 0 & \text{otherwise} \end{cases}$$

Assume that $D = 3W^2$ and that $T$ is the total cost to the customer.

(10) a. Find $E(T)$

(15) b. Find $Var(T)$.

(25) 3. Consider a population with probability mass function

$$p_X(x) = \begin{cases} \frac{1}{2} + \theta & \text{if } x = 0 \\ \frac{1}{2} - \theta & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta$ is an unknown parameter with $-\frac{1}{2} < \theta < \frac{1}{2}$. The random sample 1, 0 is drawn from this population.

(10) a. Find the method of moments estimate of $\theta$.

(15) b. Find the maximum likelihood estimate for $\theta$. Verify that it is a maximum.

(25) 4. Let $X_1$, $X_2$ be a random sample from a Poisson distribution with mean $\theta$. Consider testing $H_0$: $\theta = 5$ versus $H_a$: $\theta = 3$ with test statistic $Y = X_1 + X_2$.

(10) a. Find the probability of a Type I error if the critical region is $Y < 2$.

(15) b. Find the form of the best critical region when testing $H_0$ versus $H_a$, and simplify your answer.