Applied Statistics Comprehensive Examination

Statistical Methods I & II

1. (25 pts) A machine has a record of producing parts of which 82% are rated as excellent, 15% are rated as good and 3% are rated as unacceptable. After extensive and costly repairs to the machine, a random sample of 200 parts was obtained with 157 parts rated as excellent, 42 parts rated as good and 1 part rated as unacceptable. Decide if the repairs have changed the distribution of the output by calculating the appropriate test statistic and estimating its P-value. Be sure to provide an interpretation of the P-value and indicate how this relates to your decision.

2. (25 pts) In the production of airbag inflators for automotive safety systems, a company is interested in determining if the mean distance of the foil to the edge of the inflator is at least 2.0 cm. Suppose that past history indicates that the standard deviation of the distance measurements is 0.06 cm and that an appropriate hypothesis test will be performed using \( \alpha = 0.05 \).
   (a) (15 pts) If a sample size of 20 is used for the hypothesis test, find \( \beta \) assuming the true mean is 1.95 (that is, if \( \mu_A = 1.95 \)).
   (b) (10 pts) Find the minimum sample size required to detect a true mean of 1.95 with at least 90% power.

3. (25 pts) The government awarded grants to the agricultural departments of 9 universities to test the yield capabilities of two new varieties of wheat. Two plots of equal area were identified at each university and the two varieties of wheat were randomly assigned to these plots. The yields of the plots, in kilograms per acre, were recorded as follows:

<table>
<thead>
<tr>
<th>Variety</th>
<th>University</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>38 23 35 41 44 29 37 31 38</td>
</tr>
<tr>
<td>B</td>
<td>45 25 31 38 50 33 36 40 43</td>
</tr>
</tbody>
</table>

(a) (15 pts) Find a 95% confidence interval for the mean difference between the yields of the two varieties and use this interval to decide if there is a significant difference between the two varieties of wheat. Justify your decision.

(c) (10 pts) State the assumptions associated with this inference and indicate how you would assess them.
4. (25 pts) Four different machines are being considered for the assembling of a particular product. It is decided that 6 different operators are to be used in a randomized block experiment to compare the machines. The machines are assigned in a random order to each operator. The operation of the machines requires physical dexterity and it is expected that there will be a difference among the operators in the speed with which they operate the machines. The response variable was the amount of time (in seconds) that it took to assemble the product. The total sum of squares was 81.86 and the sums of squares for machines and operators were 15.93 and 42.09 respectively.

(a) (20 pts) Complete the ANOVA table and use it to determine if there is evidence of a significant difference among the machines using $\alpha = 0.05$.

(b) (5 pts) Decide if using operator as a blocking variable was effective in this experiment and justify your decision.